MEMO TO: Qiuyan Li

FROM: Seth Strayer

DATE: June 11, 2019

SUBJECT: TF4 – Radiation Heat Transfer

On May 5, 2019, Noah Sargent, Aaron Esquino and I conducted an experiment to verify that the total radiation emitted by a heat source is proportional to the fourth power of its absolute temperature, otherwise known as the Stefan-Boltzmann Law (Equation 1). In practice, we never deal with perfect emitters, so through this experiment we also hope to gain insight on the nature of emissivity (ϵ), which is a value between 0 (perfect absorber) and 1 (blackbody). Note that emissivity of a surface can change with temperature just as properties of a material can change, so we will examine this effect as well.

This lab utilized the Gunt Thermal Radiation Unit (WL362) to induce radiation heat transfer between the heat source and detector. This unit was connected to a multimeter which allowed us to record voltage measurements as a function of heat source temperature. The heat source was turned on and voltage measurements were taken at each 10°C increment, beginning at 30°C and ending at 150°C. Due to uncertainties in the radiation unit temperature readings, voltage measurements were taken both at the desired temperature and once more at the instant the temperature display showed 1°C above the desired temperature. Thus, a total of twenty-six voltage measurements were taken.

Given the fact that we would like to calculate irradiance [W/m^2], we had to find a calibration constant relating measured average voltage and irradiance. This calibration constant was found to be the slope of the irradiance versus voltage, using the irradiance-temperature data provided by the thermal radiation unit manufacturer (Table 1 of the TF Lab 4 Handout). I.e., a linear curve was fit between the measured averaged voltage readings and the irradiance values given in the table for each 10°C temperature point. Since zero temperature corresponds to zero irradiance, the intercept of this curve was set to zero prior to fitting.

The detector is only designed to measure incoming radiation from the heat source. Therefore, the total measured radiation (E_T), given by Equation 3, was simply the sum of the ambient radiation (E_{amb}) and the calculated heat source radiation (E_{HS}). Thus, the Stefan-Boltzmann Law could be confirmed by plotting the total measured radiation values versus heat source temperature on a log-log scale. We note that this should be a linear curve whose slope and y-intercept verifies the law. Furthermore, we may verify that the body was not a perfect emitter by calculating emissivity via comparing our experimental radiation with what we would expect from theory, given by Equation 4.

Table 1 shows a comprehensive summary of the data analysis for this experiment. Data points which are not given in this table can be found in the Excel file "TF4_Data.xlsx", sheet "Data Analysis". Following the procedure, the suggested irradiance (Table 1 of the TF Lab 4 Handout) was plotted as a function of measured average voltage. This plot is shown in Figure 1. Fitting a linear curve with a y-intercept of zero, this plot revealed a calibration constant, say k, of 67.504. Thus, irradiance was calculated for each temperature increment via multiplying the measured average voltage by the calibration constant. The total, calculated irradiance was then found by adding this value to the ambient radiation for each temperature point.

Figure 2 shows the log-log plot of irradiance vs. heat source temperature for both the experimental and theoretical calculations. We note the theoretical plot does not exactly match that as predicted by theory, indicating that there were some measurement/uncertainty errors present in our experiment. Figure 3 shows a non-log scale plot of the same data. Note that the theoretical plot is precisely that as predicted by theory; the experimental plot contains some errors, as predicted from Figure 2. Finally, Figure 4 depicts emissivity as a function of heat source temperature. We see that there is a clear trend between these two parameters; emissivity exponentially decreases as temperature increases. Thus, this behavior verifies that emissivity changes as a function of temperature just as material properties do.

Uncertainties in our measurement system play a large role in the error present in this experiment. The uncertainties of the voltage and temperature measurements were given as ±0.01 [mV] and ±0.50 [°C], respectively. Using these values, uncertainty analysis was conducted for all experimental data, given by Equations 5-10. Uncertainty in measured heat source radiation and total measured radiation was found to be constant at ±0.67504 and ±1.83613, respectively. These uncertainties propagated throughout the system and allowed us to calculate uncertainty in emissivity for each 10°C temperature point (note that these values are not constant and are listed in Table 2.

Error bars were then added to each plot based on the range of uncertainty values for each calculated value. Note that Figure 1 uses error bars in terms of uncertainty at a 95% confidence level upper and lower bounds via a regression analysis (i.e., the error bars for this plot represent the range at which we are 95% confident that the measured heat source radiation values lay between). This regression analysis is given in the Excel file “TF4_Data.xlsx”, sheet “Regression Analysis”. However, the uncertainty range was so small for each experimental calculation that they are not visible on the plots. The small range of uncertainties in this experiment indicates that there is no overlap between the experimental and theoretical values, thus indicating a significant amount of error in the experiment.

Sources of error in this experiment stem from the uncertainties in the measurement devices; these uncertainties propagate themselves throughout the system and cause our experimental values to be slightly different from those predicted by theory. We note that there is little experimental error present elsewhere, due to the trivial nature of the data collection. Although error is present, it is obvious by Figures 2 and 3 that our experimental data followed similar trends to that predicted by theory, and thus we are satisfied with the results of this experiment; in particular, Figures 2 and 3 successfully verify the Stefan-Boltzmann Law. We were also able to conclude that emissivity is present, and its value exponentially decreases as temperature increases. Future suggestions to improve the outcome of this experiment include using more precise measurement devices. Use of such devices would undoubtedly lead to less uncertainty, and thus more accurate results.

Table 1: Data Analysis

T [°C]	Average Voltage [mV]	$E_{T_{HS}}\left[\frac{W}{m^2}\right]$	$E_{amb}\left[\frac{W}{m^2}\right]$	$E_T\left[\frac{W}{m^2}\right]$	$E_{theory}\left[\frac{W}{m^2}\right]$	Emissivity	$u_{E_{T_{HS}}}\left[\frac{W}{m^2}\right]$	$u_{E_T}\left[\frac{W}{m^2}\right]$	u_ϵ [−]
30	0.345	23.29	430.28	453.57	478.87	0.95	0.67504	1.83613	0.00884
40	0.775	52.32	430.28	482.60	545.25	0.89	0.67504	1.83613	0.00788
50	1.51	101.93	430.28	532.21	618.30	0.86	0.67504	1.83613	0.00720
60	2.4	162.01	430.28	592.29	698.46	0.85	0.67504	1.83613	0.00665
70	3.38	228.16	430.28	658.45	786.17	0.84	0.67504	1.83613	0.00619
80	4.475	302.08	430.28	732.36	881.90	0.83	0.67504	1.83613	0.00580
90	5.64	380.72	430.28	811.01	986.11	0.82	0.67504	1.83613	0.00545
100	6.95	469.15	430.28	899.44	1099.30	0.82	0.67504	1.83613	0.00516
110	8.38	565.68	430.28	995.97	1221.96	0.82	0.67504	1.83613	0.00491
120	9.915	669.30	430.28	1099.59	1354.62	0.81	0.67504	1.83613	0.00468
130	11.595	782.71	430.28	1212.99	1497.79	0.81	0.67504	1.83613	0.00449
140	13.375	902.87	430.28	1333.15	1652.02	0.81	0.67504	1.83613	0.00431
150	15.315	1033.82	430.28	1464.11	1817.86	0.81	0.67504	1.83613	0.00415

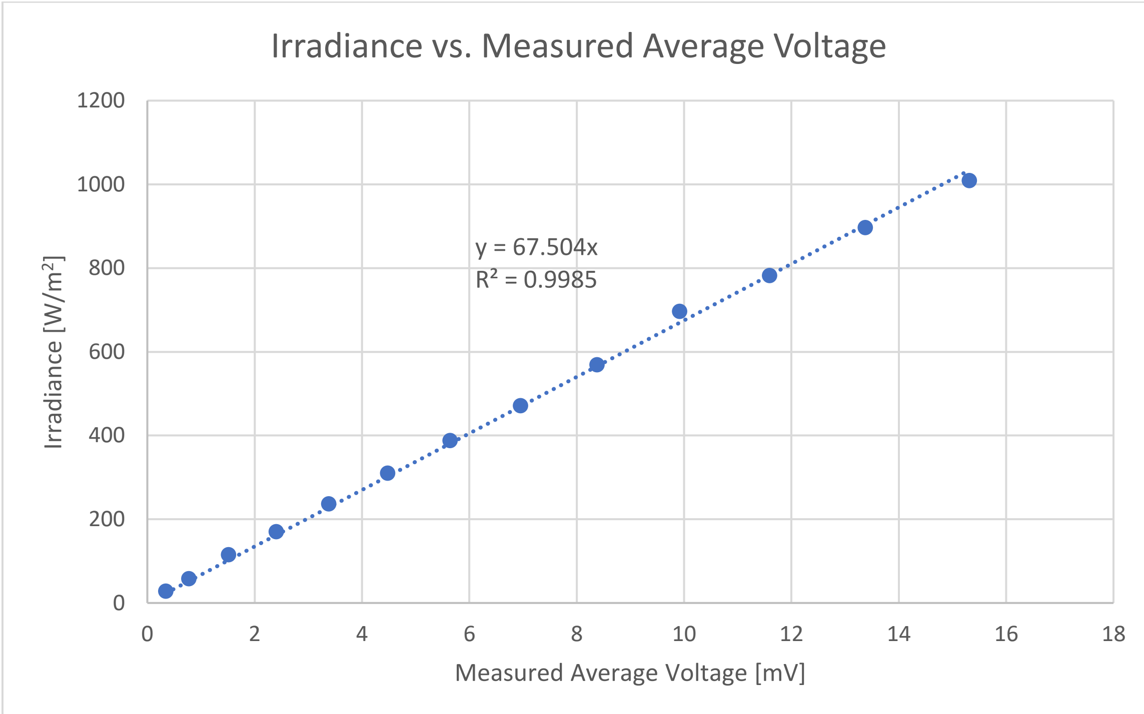

Figure 1: Irradiance vs. Measured Average Voltage

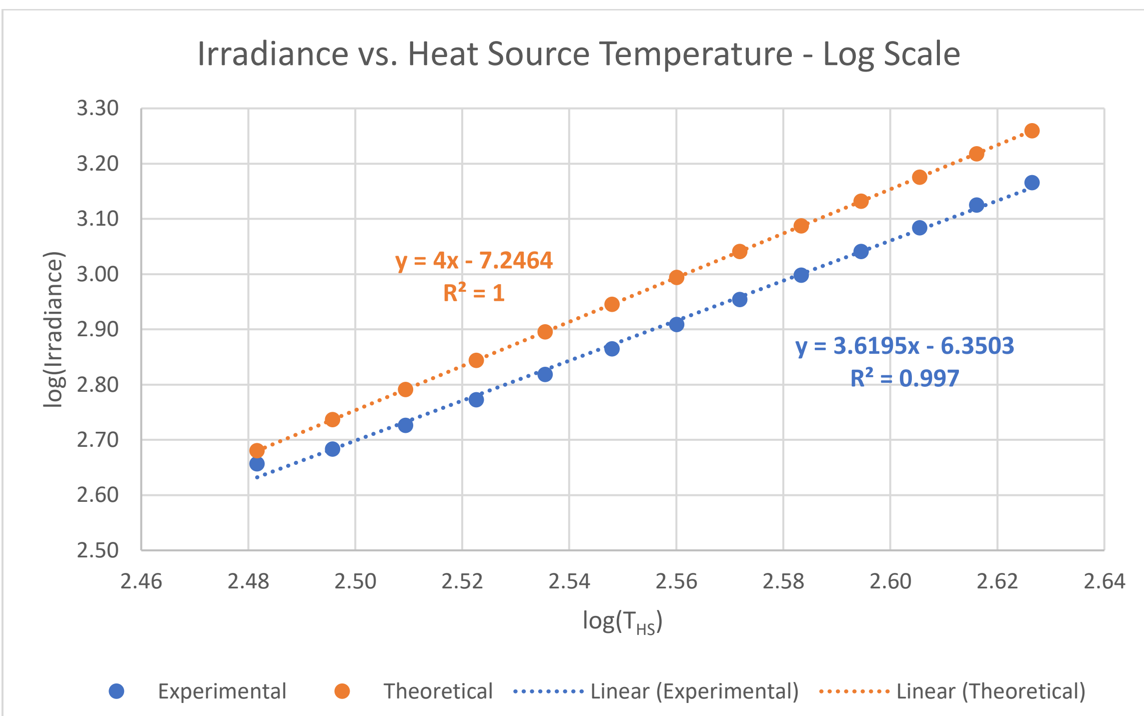

Figure 2: Irradiance vs. Heat Source Temperature – Log-log Scale

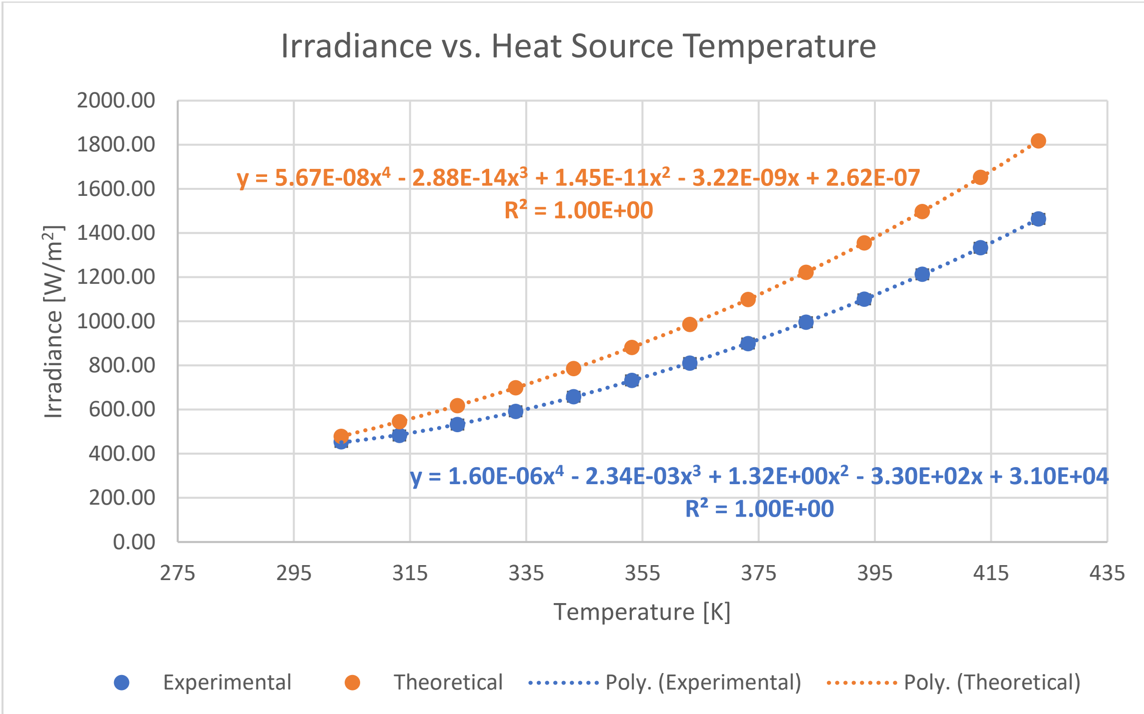

Figure 3: Irradiance vs. Heat Source Temperature

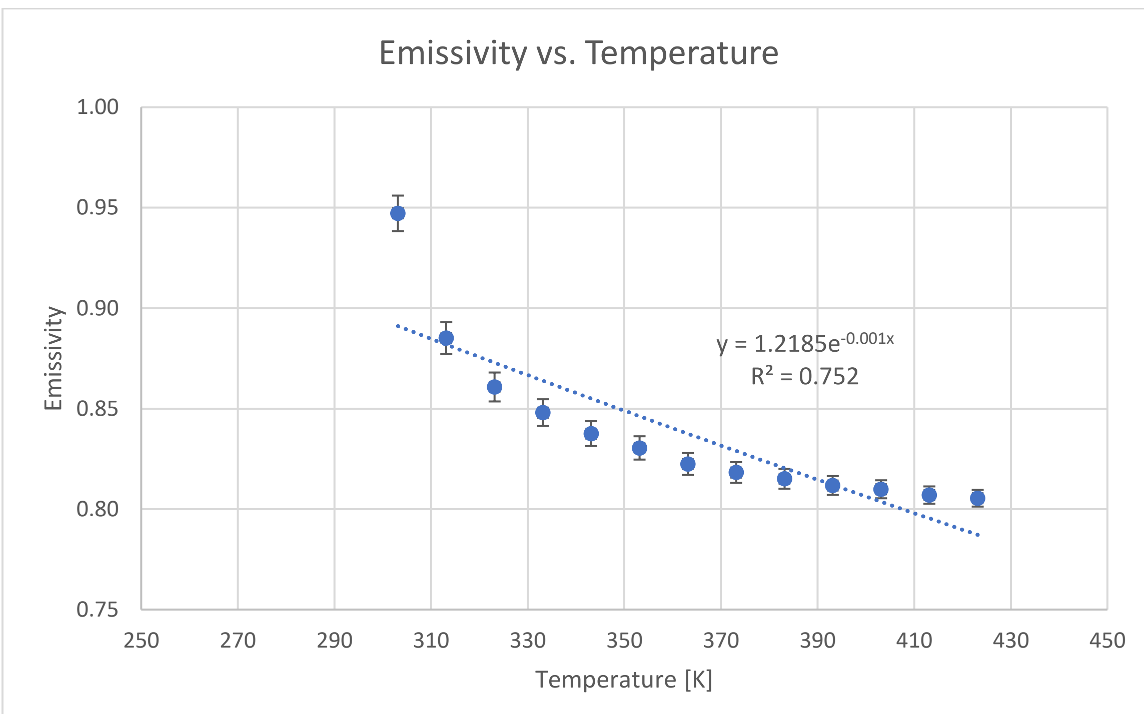

Figure 4: Emissivity vs. Temperature

Reference Equations:

The Stefan-Boltzmann Law is given by:

$$E_b = \sigma T^4 \tag{1}$$

Where E_b is the total irradiance from a blackbody, σ is the Stefan-Boltzmann constant (σ = 5.67 x 10^{-8})[1], and T is absolute temperature of the source. For an imperfect emitter, we have:

$$E = \epsilon\sigma T^4 \tag{2}$$

Where ϵ is emissivity, ranging from a value of 0 (perfect absorber) to 1 (blackbody).

The total, measured radiation for this experiment is given by:

$$E_T = E_{HS} + E_{amb} = \sigma T_{HS}^4 + \sigma T_{amb}^4 = 67.504V + \sigma T_{amb}^4 \tag{3}$$

Where E_{HS} is the measured radiation from the heat source and E_{amb} is the ambient radiation. **Note** that here, E_{HS} is calculated based on the curve fit given in Figure 1.

The emissivity for this experiment can be defined as:

$$\epsilon = \frac{E_T}{E_{theory}} = \frac{67.504V + \sigma T_{amb}^4}{\sigma T_{HS}^4} \tag{4}$$

Uncertainty Analysis:

Parameter	Value	Equation	Equation No.
Uncertainty in calculated heat source radiation	0.67504	$u_{E_{HS}} = \sqrt{(67.504u_V)^2}$	(5)
Uncertainty in total, calculated radiation	1.83613	$u_{E_T} = \sqrt{(67.504u_V)^2 + (4\sigma T_{amb}^3 u_T)^2}$	(6)
Uncertainty in emissivity	Varies with T	$u_\epsilon = \sqrt{\left(\frac{\partial\epsilon}{\partial V}u_V\right)^2 + \left(\frac{\partial\epsilon}{\partial T_{amb}}u_T\right)^2 + \left(\frac{\partial\epsilon}{\partial T_{HS}}u_T\right)^2}$	(7)
$\frac{\partial\epsilon}{\partial V}$	Varies with T	$\frac{\partial\epsilon}{\partial V} = \frac{67.504}{\sigma T_{HS}^4}$	(8)
$\frac{\partial\epsilon}{\partial T_{amb}}$	Varies with T	$\frac{\partial\epsilon}{\partial T_{amb}} = \frac{4T_{amb}^3}{T_{HS}^4}$	(9)
$\frac{\partial\epsilon}{\partial T_{HS}}$	Varies with T	$\frac{\partial\epsilon}{\partial T_{HS}} = -\frac{4(67.504)V}{\sigma T_{HS}^5} - \frac{4T_{amb}^4}{T_{HS}^5}$	(10)

References

[1] Bergman, Theodore L., et al. Fundamentals of Heat and Mass Transfer. 7th ed., J. Wiley & Sons, 2011